The Open University

Maths for science and technology

Kathleen Gilmartin, Heather Laird and Karen Rex

student toolkit 7

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Student Toolkits

Student Toolkits are produced for Open University students and cover various learning skills. Each is designed as a resource to help you to engage with your studies; the materials they contain are not prescriptive, *you* decide when and how to use them.

- Student Toolkit 1 *The effective use of English*
- Student Toolkit 2 *Revision and examinations*
- Student Toolkit 3 *Working with charts, graphs and tables*
- Student Toolkit 4 *Reading and note taking*
- Student Toolkit 5 *Essay and report writing skills*
- Student Toolkit 6 *More charts, graphs and tables*
- Student Toolkit 7 *Maths for science and technology*
- Student Toolkit 8 *Learning how to learn*
- Student Toolkit 9 *Extending and developing your thinking skills*
- Student Toolkit 10 *Using a computer for study*
- Student Toolkit 11 *Giving presentations*

Copies of these are available from your Regional Centre.

1 Introduction

How do you feel about the mathematics in your course materials? It is quite possible that you do not feel as confident as you would like. If you have not done any mathematics recently, you may feel that you need to refresh your memory or there may be a specific topic that you never really fully understood.

Other students have said:

> *When I'm faced with a triangle, I never know which trig ratio to use.*
>
> *I'm alright until I get to the point where a question asks me to use logs to solve a problem, then my mind goes blank.*

This booklet is one in a series of Student Toolkits; there are others to help you with such things as note taking, essay and report writing, effective use of English, revision and examinations, and working with charts, graphs and tables. Your regional Student Services will be able to tell you which others are available and would be appropriate for your needs.

The mathematical skills assumed by Open University courses in the Faculty of Mathematics and Computing, Faculty of Science and Faculty of Technology, vary greatly from course to course. Students are strongly recommended to start by reading the *Sciences Good Study Guide* as preparation for whichever course they are going to study. This guide is an excellent place to start but you may have found that the section on maths does not go far enough for you. Equally, you may have discovered that you don't feel completely confident about some of the maths skills assumed by your course materials. This booklet aims to follow on from where the *Sciences Good Study Guide* stops and covers the following topics:

- indices
- solving equations
- scientific notation, units and significant figures
- basic trigonometry
- logarithms.

This is a practical booklet. It begins in Section 2 by asking you to reflect on why you decided to request this booklet. It then goes on to explore which skills you need help with by focussing on your course materials and the assessment questions. Sections 3–7 then explain how these skills are used and, in some cases, why they are used. In Section 8, there are some suggestions for further reading and other sources of help. It is not essential that you work through each section of the booklet, you may just want to read the section or sections that address your personal areas of interest.

We expect you to be an active learner and, as with your course material, we suggest that you do the activities as you go. You will be able to see where we ask you to do some work by the symbol of a pencil in the margin.

We have tried to provide as many opportunities for you to practise your skills as possible because we believe that this is the best way to gain confidence in doing mathematics. Remember that it often takes time to master a skill completely, so you may want to refer back to this booklet from time to time in the future. If this booklet gives you a better understanding of some of the mathematical techniques used in your course, then it will have achieved its aim.

2 Reflection on mathematics

As you have seen from the Introduction, we have designed this toolkit for students on mathematics, technology and science courses who feel the need to get some more help with the mathematical content of their courses. We imagine a common reason why students will ask for a copy of this toolkit is because of a lack of confidence in their mathematical skills at the level that they feel will be needed in their course. However, we do not want to take this for granted. Therefore, we have written this section to help you to look at your reasons for requesting the toolkit, so that you can make plans to improve in one or more areas of mathematics.

Activity 1

Before you start to work through the rest of the booklet, spend a few minutes thinking about why you felt that this toolkit might be helpful. In particular, answer the following questions:

- What made me decide to look for help?
- Why did this toolkit sound as if it would be useful?
- What do I hope to gain by using this toolkit?

Don't spend too long on answering these questions. We suggest that you write down the answers before you look at our suggestions, and then keep your answers to return to later after you have done some work on the toolkit. Brief notes are enough. It should take you no more than 15–20 minutes *at most*.

Discussion

The ideas below may be similar to yours or you may find them useful to add to your own. Your answers may be completely different, of course, and we are not suggesting that ours are the 'right' ones – but we hope that they might act as a trigger for you to think about other things.

- What made me decide to look for help?

There are, of course, many reasons why students decide to ask for help. It may be that you have read other Student Toolkits designed to help those with concerns about mathematics and found them useful. It may be that you feel that you can understand a little about mathematics, but you are concerned that this is not enough for your current course. We have spoken to other students, and asked them this question, and some of their responses are given below.

> *Since school, I needed maths in my studies, and managed to get a smattering of statistics, etc. I am actually numerate by comparison with many of my colleagues. I enjoy financial planning (of my own affairs) and cope with managing budgets at work. However, I still avoid work that involves a lot of figures.*

> *I am constantly worried about the use of maths – I always presume that I will get simple problems wrong if numbers are involved.*

> *I have looked at the set book – the recommended reading is downright scary. I am just grateful that there is not an exam to do because I would never be able to pass at this level on my own.*

- Why did this toolkit sound as if it would be useful?

This one may be more difficult to answer. It depends on your ability and confidence in numerical work. You may feel that you need all the help that you can get and simply hope that this will be useful in some way that you can't yet define. We believe that, the more that you can decide on the answer to this sort of question at this stage, the more likely you are to gain from using this toolkit.

- What do I hope to gain by using this toolkit?

This depends very much on the reasons why you asked for this toolkit. However, you might have included things like the students quoted below.

> *The skills I have needed as a student with the OU have been to do with interpreting information from research – both numeric and written – and judging the validity and reliability. I always shy away from quantitative research.*
>
> *I have been studying for several years now, starting with 'O' levels (as was), 'A' levels and, more recently the OU. The three courses I have completed are two in biology and one in chemistry. In both subjects, the unfamiliar units used gave me pause for thought.*
>
> *I am not very confident with algebra, and I know that I will need to learn how to do it for the next TMA.*

One reason that you may be interested in this toolkit is that you realize that your course requires you to carry out some numerical calculations, and you don't feel very confident with this. The next activity asks you to think about the use of mathematics in your current piece of assessed work. If you haven't started your course yet for this year, then carry on to the next sections, and come back to Activity 2 in a couple of months.

Activity 2 (Part 1)

Courses vary enormously in terms of their mathematical content and we suggest that you spend some time looking at your own course now. We hope that you will gain an idea of the sort of mathematical knowledge needed for your course. This should help you to plan what you might need to improve. (You may be able to find overall details in the Course Guide.) Look at the assessment that is due next for the course that you are studying. It may include a TMA, a CMA, an examinable component or an exam, depending on what stage you have reached in the course. Identify from this the parts of the assessment that require numerical skills and knowledge.

- What are these skills?
- What do I have to understand before I can tackle the assessment?
- Where can I get help with these?

Discussion

- What are these skills?

Look at the questions that the assessment asks you to answer. This should enable you to make a list of what is being assessed at this time. You can repeat this exercise on a regular basis, each time you find that you have a need to identify the mathematics involved in your work.

- What do I have to understand before I can tackle the assessment?

Look at the unit or block of your course that is covered by the next piece of assessment, and try to identify what the learning outcomes of that part of the course are. For example, they may say that you will be able to understand a particular concept, that you will be able to understand and use algebra or scientific notation. Different courses will have different layouts for the outcomes. You may be able to find them at the end of units, in the Assignment Booklet, in the Course Guide or in separate block guides. Once you have found the outcomes, decide which are to do with mathematical understanding. Now make a list of the mathematics that you are being asked to learn. If you aren't sure about what is required or if numerical work is not mentioned explicitly, you might need to clarify this with your tutor. The Course Team may have assumed that you have some skills already, but your tutor will know this.

- Where can I get help with these?

Well, that will vary, depending on your course. You may find that your course has some teaching about mathematics built into it, or it may refer you to some extra materials that you can get hold of. Tutorials, regional day schools or residential schools may be helpful, if you are able to attend them. You can also contact your tutor directly. In addition, we hope that you will find the relevant sections of this toolkit useful. We have also included a list of reference materials at the end of the toolkit.

You should now have two lists, one that says what mathematics is assessed and one that says what is taught. If you aren't sure about these, then you may need to either look at the block or unit in more detail, or talk to your tutor or other students. You could perhaps check this out through a FirstClass conference, if your course has one.

Activity 2 (Part 2)

Once you have made these lists, you need to decide on priorities. We suggest that you:

- decide the areas that you need to learn
- make a list of these areas in order of priority
- decide to tackle the one that you need to do in the short term as the first priority, then move on to others
- check the rest of this toolkit, to see if what you need is covered directly
- identify areas not covered here and look at the list of other resources at the back of the toolkit.

Discussion

Once you have made a list of those mathematical aspects you need to learn, don't be discouraged if there seems to be quite a lot that you need to do. This can be off-putting, and you might feel that you want to give up, but by preparing a prioritized list you should be able to see what you need to tackle most urgently, and make a plan for that first. Then you can come back to the rest when you need to.

This toolkit may directly cover your priority areas. If it does, then we suggest that you turn to the relevant parts in Sections 3–7. If it doesn't, try the list in Section 8 'Further reading and sources of help'.

3 Indices

In mathematics, we often need to find a shorthand way of representing information or data. Nowhere is this need more obvious than when we wish to represent something like the product of 2 multiplied by itself 2, 6, 10, 15 or even 20 times.

Instead of writing $2 \times 2 \times 2 \times 2 \times 2 \times 2$, we write 2^6. This is read (and said) as '2 to the power 6'; 6 is the index of the power. In general, this means that

$a^n = a \times a \times a \times a \times \ldots\ldots$ to n factors

where n is called the index.

Both a and n can be either positive or negative numbers; a^{-n} can also be written as $\frac{1}{a^n}$.

From this simple definition, we can now go on to look at power notation, or index notation as it is often called, in more detail. We will establish the five rules that are used with this notation.

Rule 1 $a^m \times a^n = a^{(m+n)}$

$a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a^5$

Thus $a^3 \times a^2 = a^{(3+2)} = a^5$

This rule works for both positive and negative indices.

$$a^{-3} \times a^5 = \frac{1}{a \times a \times a} \times a \times a \times a \times a \times a = \frac{a \times a \times a \times a \times a}{a \times a \times a} = a \times a = a^2$$

Thus $a^{-3} \times a^5 = a^{(-3+5)} = a^2$

Rule 2 $a^m \div a^n = a^{(m-n)}$

$$a^5 \div a^2 = \frac{a \times a \times a \times a \times a}{a \times a} = a \times a \times a = a^3$$

Thus $a^5 \div a^2 = a^{(5-2)} = a^3$

This rule works for positive and negative indices, but you need to take care when dividing by a negative power.

$$a^2 \div a^{-6} = a \times a \div \frac{1}{a \times a \times a \times a \times a \times a} = (a \times a) \times (a \times a \times a \times a \times a \times a) = a^8$$

Thus $a^2 \div a^{-6} = a^{(2-(-6))} = a^8$

Rules 1 and 2 only work if the powers involve the same factor, for example, you can't simplify $a^3 \times b^2$ or $a^7 - b^5$.

Rule 3 $a^0 = 1$

This is because

$a \div a = 1$ $\quad a^3 \div a^3 = 1$ $\quad a^{10} \div a^{10} = 1$ $\quad$ and $\quad a^n \div a^n = 1$

and, using the rule for division (Rule 2),

$a^n \div a^n = a^{(n-n)} = a^0 = 1$

Remember $a = a^1$

Note that we can now explain why $a^{-n} = \frac{1}{a^n}$:

a^{-n} can be written as $a^{(0-n)}$

and, using Rule 2, this is the same as $a^0 \div a^n$

which, using Rule 3, is the same as

$1 \div a^n = \frac{1}{a^n}$

In the same way, we can show that $a^n = \frac{1}{a^{-n}}$:

$\frac{1}{a^{-n}}$ can be written as $1 \div a^n$

so, using Rule 2 and Rule 3,

$\frac{1}{a^{-n}} = a^0 \div a^{-n} = a^{(0-(-n))} = a^n$

Rule 4 $(a^m)^n = a^{mn}$

We know that $a^2 = a \times a$

So, $(a^3)^2 = a^3 \times a^3 = a^{(3+3)} = a^6 = a^{(3 \times 2)}$

This rule works for positive and negative indices.

For example, $(a^{-4})^3 = a^{-12}$

Rule 5 $a^{p/q} = \sqrt[q]{a^p}$

$(a^{1/2})^2 = a^{1/2} \times a^{1/2} = a^1 = a$

$(a^{1/3})^3 = a^{1/3} \times a^{1/3} \times a^{1/3} = a^1 = a$

$(a^{1/4})^4 = a^{1/4} \times a^{1/4} \times a^{1/4} \times a^{1/4} = a^1 = a$

Since $a^{1/2}$ squared is equal to a, $a^{1/2}$ is the square root of a and is sometimes written as $\sqrt{a}$.

$a^{1/3}$ cubed is equal to a, so $a^{1/3}$ is the cube root of a and can be written as $\sqrt[3]{a}$.

$a^{1/4}$ to the power of 4 is equal to a, which means that $a^{1/4}$ is the fourth root of a, written as $\sqrt[4]{a}$.

So it follows that

$a^{1/n}$ is the nth root of a and can be written as $\sqrt[n]{a}$.

If we use Rule 4 to write $a^{p/q}$ as $(a^p)^{1/q}$ or $(a^{1/q})^p$, you should now be able to see that $a^{p/q}$ is the qth root of a^p which is written as $\sqrt[q]{a^p}$.

Rules 1, 2, 4 and 5 all work with fractional indices.

Activity 1

Simplify each of the following:

(a) $4^{3/2}$

(b) $125^{-2/3}$

(c) $4a^2b^3 \times 5a^5b^4$

(d) $48a^8 \div 6a^4$

(e) $3a^3 \div 12a^9$

(f) $\frac{a^9 \times a^3}{a^6}$

(g) $\left[\frac{a^5 b}{3a^2}\right]^3$

(h) $\left[\frac{16a^6}{9a^4}\right]^{1/2}$

Discussion

(a) $4^{3/2} = (4^{1/2})^3$ which can be written as $(\sqrt{4})^3 = 2^3 = 8$

(b) $125^{-2/3} = \frac{1}{125^{2/3}} = \frac{1}{(125^{1/3})^2} = \frac{1}{5^2} = \frac{1}{25}$

You might like to check the answers to (a) and (b) using a calculator.

(c) $4a^2b^3 \times 5a^5b^4 = 20a^{(2+5)}b^{(3+4)} = 20a^7b^7$ which can be written as $20(ab)^7$

(d) $48a^8 \div 6a^4 = 8a^{(8-4)} = 8a^4$

(e) $3a^3 \div 12a^9 = \dfrac{a^{(3-9)}}{4} = \dfrac{a^{-6}}{4} = \dfrac{1}{4a^6}$

(f) $\dfrac{a^9 \times a^3}{a^6} = a^{(9+3)} \div a^6 = a^{(9+3-6)} = a^6$

(g) $\left[\dfrac{a^5 b}{3a^2}\right]^3 = \left[\dfrac{a^{(5-2)} b}{3}\right]^3 = \left[\dfrac{a^3 b}{3}\right]^3 = \dfrac{a^9 b^3}{27}$

(h) $\left[\dfrac{16a^6}{9a^4}\right]^{1/2} = \left[\dfrac{16a^{(6-4)}}{9}\right]^{1/2} = \left[\dfrac{16a^2}{9}\right]^{1/2} = \dfrac{4a}{3}$

Activity 2

Simplify:

(a) $\dfrac{2^n \times 4^{n+2}}{8^{n-1}}$

(b) $(8^{1/2} \times 3^{3/2})^{2/3}$

(c) $\dfrac{8^{n+2} - 6(2^{3n+3})}{2^n \times 4^{n+2}}$

Discussion

(a) $\dfrac{2^n \times 4^{n+2}}{8^{n-1}} = \dfrac{2^n \times (2^2)^{n+2}}{(2^3)^{n-1}}$ Rule 4

$= \dfrac{2^n \times 2^{2n+4}}{2^{3n-3}}$

$= \dfrac{2^{(n+2n+4)}}{2^{3n-3}}$ Rule 1

$= \dfrac{2^{3n+4}}{2^{3n-3}}$

$= 2^{3n+4-(3n-3)}$ Rule 2

$= 2^{3n+4-3n+3} = 2^7 = 128$

(b) $(8^{1/2} \times 3^{3/2})^{2/3} = ((2^3)^{1/2} \times 3^{3/2})^{2/3}$

$= (2^{3/2} \times 3^{3/2})^{2/3}$ Rule 4

$= 2^{(3/2 \times 2/3)} \times 3^{(3/2 \times 2/3)}$ Rule 4

$= 2^1 \times 3^1 = 6$

(c) $\dfrac{8^{n+2} - 6(2^{3n+3})}{2^n \times 4^{n+2}} = \dfrac{(2^3)^{n+2} - 3 \times 2 \times 2^{3n+3}}{2^n \times (2^2)^{n+2}}$

$= \dfrac{2^{3n+6} - 3(2^{3n+4})}{2^n \times 2^{2n+4}}$ Rule 1, 4

$= \dfrac{2^2(2^{3n+4}) - 3(2^{3n+4})}{2^{3n+4}}$ Rule 1

$= \dfrac{2^{3n+4}(2^2 - 3)}{2^{3n+4}}$ Rule 1

$= 2^2 - 3 = 1$

Although you may arrive at the answer to an activity in a different way from that given in this toolkit (because the rules of indices can often be applied in a different order), you should always get the same final answer.

All the examples we have discussed in this section have a simple answer, but this will not always be the case, particularly where the calculation relates to a situation modelled on the real world.

4 Equations and algebra

In mathematics, equations are used a great deal in arriving at a solution to a problem. In this section, we wish to refresh your memory of how you can solve an equation using algebraic techniques. We have assumed that you have prior knowledge of algebra and need reintroducing to it, rather than are starting from scratch.

This section is in five parts.

A Why use algebra?

This is a brief explanation of why algebraic techniques are useful.

B Simplifying algebraic expressions

You should always try to simplify expressions and equations as much as possible, to make them easier to use.

C Changing the subject of an equation

This section shows you how to express some algebraic equations in terms of the others.

D Solving simple equations with one unknown

This section shows you how to find the solution where there is one unknown, using algebra.

E Simultaneous equations

This section shows you how to find the solutions where you have pairs of equations, each with the same two unknowns.

In each of parts B to E, we will outline the techniques, talk you through an example, and then give you some exercises in activities to do for yourself. Answers to the activities are together at the end of this section.

A Why use algebra?

We've seen examples in Section 2 where the quantities in problems are always related in a particular way. For example, $V = u + at$ is a general expression connecting initial and final velocity (u and V), constant acceleration (a) and time (t). This single formula allows you to calculate final velocity, V, for any values of u, a and t, not just for one specific set of data. Algebra enables us to generalize – something may seem to be true for many sets of specific data values, but is it true for all such sets?

Before discussing equations in more detail, we would like to show you what we mean.

Example 1

Find the sum of three consecutive whole numbers or integers. Divide your answer by 3. What do you notice? (You may need to try this for several sets until you find a pattern.)

We can explain this by working out a formula for this sum. Suppose we call the first number n, what will the second and third numbers be? What will their sum be, in terms of n? Can you now explain the result?

Discussion

The three consecutive numbers are n, $(n + 1)$ and $(n + 2)$. If you add these together, you get $n + (n + 1) + (n + 2) = 3n + 3$.

Dividing by 3 gives $\frac{3n + 3}{3} = n + 1$.

Therefore, the sum of three consecutive numbers is always divisible by three.

B Simplifying algebraic expressions

Whenever we wish to solve an expression, we should first rearrange it to make it as simple as possible, then manipulate it. If we try to manipulate expressions in a complex form, we increase the possibility of making mistakes or becoming confused. Once the expression has been simplified, we can either solve it or state it in its most simplified form.

Sequence of arithmetical calculations

Brackets are used to avoid any ambiguity in a calculation. Any calculations inside a bracket are usually carried out first.

For example, $2 \times (7 + 4) = 2 \times 11 = 22$ *or* $2 \times 7 + 2 \times 4 = 22$

Without brackets, $2 \times 7 + 4$ *could* mean either $(2 \times 7) + 4$ *or* $2 \times (7 + 4)$, but see below for conventions on which operations are done first.

Similarly, $2 – (8 – 9)$ is not the same as $2 – 8 – 9$:

$2 – (8 – 9) = 2 – (–1) = 2 + 1 = 3$ *or* $2 – 8 + 9 = 3$

whereas $2 – 8 – 9 = 2 – 17 = –15$

Algebra has rules and conventions in the way that it operates. In other words, there are things that you have to do (rules) and things that mean that mathematics is presented consistently (conventions). We saw examples of rules in Section 3 on indices. Conventionally, when there are no brackets, multiplication and division are done before addition and subtraction; most calculators will do this automatically.

For example:

(a) $5 \times 8 + 7 = (5 \times 8) + 7 = 40 + 7 = 47$ *not* $5 \times (8 + 7) = 5 \times 15 = 75$

(b) $8 \div 4 + 9 = (8 \div 4) + 9 = 2 + 9 = 11$ *not* $8 \div (4 + 9) = 8 \div 13$

(c) $5 \times 4 – 12 \div 3 + 7 = (5 \times 4) – (12 \div 3) + 7 = 20 – 4 + 7= 27 – 4 = 23$

Other useful conventions are:

- writing $4n$ instead of $4 \times n$; $4mn$ is $4 \times m \times n$
- writing $4n$ rather than $n4$
- writing the product of multiplying x by itself as x^2 instead of xx
- writing x rather than $1x$.

We will now look at some examples of simplifying expressions.

Example 2

Simplify:

$12t + 13t^2 - 4 - 6t + 3t^2$

Discussion

There are three kinds of terms in this expression. Simple numbers like 4, terms like $12t$ and terms like $13t^2$.

Two useful rules follow:

- when adding like terms, add the coefficients; for example, $2x + 5x = 7x$
- avoid mixing powers of the same variable – you can't add x and x^2 to give a single term, as they represent different numbers; for example, when $x = 2$, $x^2 = 4$.

By convention, the terms with the highest power go at the start of the expression.

So, start by collecting all the terms that relate to t and t^2 together, and begin the statement with the terms that relate to t^2.

$12t + 13t^2 - 4 - 6t + 3t^2 = 13t^2 + 3t^2 + 12t - 6t - 4$

Now, we will add these terms together, and simplify as far as possible.

$13t^2 + 3t^2 + 12t - 6t - 4 = 16t^2 + 6t - 4 = 2(8t^2 + 3t - 2)$

Activity 1

Simplify the following.

(a) $2t^2 + 4t^2$

(b) $6y - 8y^2 + 4y^2 - 2y$

(c) $4c^3 + 2c^2d - 4 - c^2d$

(d) $12 + 2r^2 - 4rs - 3r^2 + 5rs$

(e) $\dfrac{a^2}{2} + 3a^2 + a - a^2 + \dfrac{a}{4}$

Discussion

See page 17 (but do try these before you look at the answers).

We can now go on to discuss further the use of brackets in mathematics. As you have seen, brackets are used to indicate the order in which a numerical calculation should be carried out. This is also true when you are using symbols.

So, for example, $2(2n + 5) = 4n + 10$

As a convention, $2(2n + 5)$ is preferable to $(2n + 5)2$, although, since it doesn't matter what order you multiply two numbers in, they are equivalent.

To multiply out brackets, the critical factor is that all of the expression inside the brackets is multiplied by the figure outside them.

So, for example, $x - 2(x - 3) = x - 2 \times x - 2 \times -3 = x - 2x + 6 = -x + 6$

If we are multiplying two expressions within brackets together, such as $(x + 2)(x - 1)$, we can approach this in several ways. The most common is to split the first bracket out, so

$(x + 2)\ (x - 1) = x(x - 1) + 2(x - 1) = x^2 - x + 2x - 2 = x^2 + x - 2$

Activity 2

Expand each of the following expressions by removing the brackets. Then (if possible) collect like terms.

(a) $2(2x - y)$

(b) $-2(a + 2b - c)$

(c) $4(r + 6s) - (4s - r)$

(d) $x(2x - 3) - 2x(5 - 2x)$

(e) $(x - 2)(x + 3)$

(f) $(r + t)^2$

(g) $(a - 4)(a + 4)$

(h) $\frac{1}{3}(p - 6)^2$

Discussion

See page 17.

Activity 3

Simplify the following.

(a) $1 - (2 - x) + 2x$

(b) $1 - (2 - ((3 - x) + 2x)) + 3x$

(c) $1 - (2 - (3 - (4 - x) + 2x) + 3x) + 4x$

Discussion

See page 17.

C Changing the subject of an equation

Now, we will look at rearranging an equation. Before we do this, we would like you to look at some general rules for adding, subtracting, multiplying and dividing positive and negative numbers. Symbols like a or x are simply 'standing in' for numerical values, so where a term is positive or negative, it should be treated in the same way as the rules below.

General rules

Positive and negative numbers

Numbers which have a plus sign attached to them, such as +7, are referred to as positive numbers. Numbers which have a minus sign attached to them, such as –5, are referred to as negative numbers.

Addition of numbers with the same sign

When adding numbers with the same sign, the sign of the sum is the same as the sign on each of the numbers.

$-5 + (-4) = -9$

$+5 + (+9) = +14$

When adding numbers with the same sign, you can omit the brackets and the + sign for the addition. When the first number is positive, it is usual to omit its + sign.

$+5 + (+9) = +14$ can be written as $5 + 9 = 14$

$-5 + (-4) = -9$ can be written as $-5 - 4 = -9$

Addition of numbers with different signs

To add numbers whose signs are different, subtract the numerically smaller from the larger. The sign of the result is the same as the sign of the numerically larger number.

$-12 + 6 = -6$

$11 - 16 = -5$

When dealing with several numbers of different signs, separately add the positive and negative numbers together. The set of numbers then becomes two numbers, one positive and one negative, which you can add in the usual way.

$-16 +11 - 7 +3 + 8 = - 23 + 22 = -1$

Subtraction

To subtract numbers, change the sign of the number being subtracted and add the resulting number.

$-4 - (+7) = -4 + (-7) = -4 - 7= -11$

$+3 - (-10) = +3 + 10 = +13$

Multiplication

The product of two numbers with similar signs is positive, while the product of two numbers with different signs is negative.

$(-3) \times (-5) = +15$	Negative × negative = positive
$(+3) \times (-5) = -15$	Positive × negative = negative

If you multiply any number by zero, the answer is always zero.

Division

When dividing, numbers with similar signs give a positive answer and numbers with different signs give a negative answer.

$-15 \div 3 = \frac{-15}{3} = -5$

$-15 \div -3 = \frac{-15}{-3} = 5$

If you divide a number by zero, you will always get an error, as this is impossible.

Changing the subject of an equation

We may wish to rearrange an equation, to change its subject. Some equations may have more than one algebraic symbol that is an unknown. The equation may be expressed in terms of x, say $x = y + 4$. We may wish to know what the equation would look like if it were expressed in terms of y. The critical thing with any equation is that you must always carry out the same calculations to each side of the equation. If, for example, you deduct x from one side of the equation but not from the other, it changes the equation completely. Since the quantities on each side of an equation are the same, anything done to one side must be done to the other to maintain the equality.

Say we wish to rearrange $x = y + 3$ so that y becomes the subject of the equation.

We can subtract 3 from both sides of the equation, and simplify.

$x = y + 3$

$x - 3 = y + 3 - 3$

$x - 3 = y$

As another example, say we wish to rearrange $x = \frac{y}{5} - 2$ so that y becomes the subject of the equation.

First, we multiply both sides by 5: $5x = y - 10$

Then we add 10 to both sides: $5x + 10 = y$

Activity 4

Now try some rearrangement exercises for yourself. In all cases, make x the subject of the equation.

(a) $x - 5 = y$

(b) $3x - 3 = y$

(c) $\frac{x}{2} = 3y$

(d) $\frac{x}{7} - 4 = 2y$

(e) $\frac{x}{2.5} = 10y$

(f) $2x - 12 = 2y$

Discussion

See page 18.

D Solving simple equations with one unknown

Now we wish to move on to equations where you can find a solution. The work that you have done so far on simplifying and rearranging equations will be used here.

Example 3

Consider the equation $4x = 24$. We call x the unknown and we want to find the value of x which makes the statement $4x = 24$ true.

Discussion

You may be able to see this straight away, but if not, divide both sides of the equation by 4, to get an answer for x.

$4x = 24$

$\frac{4x}{4} = \frac{24}{4}$

The solution is, therefore, $x = 6$.

You can check the accuracy of your answer by substituting back into the original equation

$4 \times 6 = 24$.

Activity 5

Now solve the following equations for x.

(a) $3x = 6$

(b) $7x = 21$

(c) $8x = 32$

(d) $x + 3 = 9$

(e) $x + 6 = 7$

(f) $x + 7 = 11$

Discussion

See page 18.

We would now like you to do some slightly more complex equations, which involve rearrangements and multiplying out of brackets.

Activity 6

Now solve the following equations for x.

(a) $2x - 1 = 7$

(b) $5x - 8 = 2$

(c) $3x + 8 = 5$

(d) $3x - 8 = 5x - 20$

(e) $3(x + 1) = 9$

(f) $23 - x = x + 11$

(g) $2(x - 3) - (x - 2) = 5$

(h) $\frac{4x}{5} = 12$

(i) $\frac{x}{3} = \frac{2}{5}$

(j) $\frac{x}{3} + \frac{x}{5} = 2$

(k) $\frac{x}{4} - \frac{x}{7} = 2$

Discussion

See page 18.

E Simultaneous equations

Simultaneous equations are pairs of equations that are both true (i.e. they are simultaneously true). They are both expressed as equations with two unknowns. By making one of these unknowns the subject of both equations, we can then substitute the subject in one equation and then solve for the other unknown. Then we can substitute back into the equation and solve for the subject.

Example 4

We can show you what we mean with this example.

$2y = x + 7$

$y = x + 2$

Discussion

First, we need to rearrange these equations to make either x or y the subject. We have chosen to make x the subject. The order doesn't matter, as the answers that you will get must be the same.

$2y = x + 7$	deduct 7 from both sides	$2y - 7 = x + 7 - 7$	So, $2y - 7 = x$
$y = x + 2$	deduct 2 from both sides	$y - 2 = x + 2 - 2$	So, $y - 2 = x$

Therefore, $x = 2y - 7$ and $x = y - 2$

So, $2y - 7 = y - 2$	deduct y from both sides	$2y - y - 7 = y - y - 2$
So, $y - 7 = -2$	add 7 to both sides	$y - 7 + 7 = -2 + 7$

Therefore, $y = 5$

We can substitute this back into either of the original equations.

$2y = x + 7$

So, $10 = x + 7$

Therefore, $x = 3$

As a check, make sure that this is true of the other equation.

$x + 2 = y$

$x = y - 2 = 5 - 2 = 3$

So the solution is $x = 3$, $y = 5$. Finally, check that these values satisfy both the original equations.

Activity 7

Now try to solve the following pairs of simultaneous equations for yourself.

(a) $y = x + 10$ $\quad 3y = 2x + 5$

(b) $y = 4x$ $\quad y = 3x + 5$

(c) $y = 7x + 4$ $\quad 3y = x + 7$

(d) $y = 2x + 4$ $\quad 3y = x + 7$

Discussion

See page 19.

Solutions to the activities

Activity 1

Discussion

(a) $2t^2 + 4t^2 = 6t^2$

(b) $6y - 8y^2 + 4y^2 - 2y = -8y^2 + 4y^2 + 6y - 2y = -4y^2 + 4y$

(c) $4c^3 + 2c^2d - 4 - c^2d = 4c^3 + 2c^2d - c^2d - 4 = 4c^3 + c^2d - 4$

(d) $12 + 2r^2 - 4rs - 3r^2 + 5rs = 2r^2 - 3r^2 - 4rs + 5rs + 12 = -r^2 + rs + 12$

(e) $\frac{a^2}{2} + 3a^2 + a - a^2 + \frac{a}{4} = 3a^2 - a^2 + \frac{a^2}{2} + a + \frac{a}{4} = \frac{5a^2}{2} + \frac{5a}{4}$

Activity 2

Discussion

(a) $2(2x - y) = 4x - 2y$

(b) $-2(a + 2b - c) = -2a - 4b + 2c$

(c) $4(r + 6s) - (4s - r) = 4r + 24s - 4s + r = 5r + 20s$

(d) $x(2x - 3) - 2x(5 - 2x) = 2x^2 - 3x - 10x + 4x^2 = 6x^2 - 13x$

(e) $(x - 2)\,(x + 3) = x^2 - 2x + 3x - 6 = x^2 + x - 6$

(f) $(r + t)^2 = (r + t)\,(r + t) = r^2 + rt + tr + t^2 = r^2 + t^2 + 2rt$

(g) $(a - 4)(a + 4) = a^2 - 4a + 4a - 16 = a^2 - 16$

(h) $\frac{1}{3}(p - 6)^2 = \frac{1}{3}(p - 6)\,(p - 6) = \frac{1}{3}(p^2 - 6p - 6p + 36) = \frac{1}{3}(p^2 - 12p + 36)$

$= \frac{1}{3}p^2 - 4p + 12$

Activity 3

Discussion

(a) $1 - (2 - x) + 2x = -1 + x + 2x = 3x - 1$

(b) $1 - (2 - ((3 - x) + 2x)) + 3x = 1 - (2 - (3 + x)) + 3x$

$= 1 - (2 - 3 - x) + 3x$

$= 1 - (-1 - x) + 3x$

$= 1 + 1 + x + 3x$

$= 2 + 4x$

(c) $1 - (2 - (3 - (4 - x) + 2x) + 3x) + 4x = 1 - (2 - (3 - 4 + x + 2x) + 3x) + 4x$

$= 1 - (2 - (-1 + 3x) + 3x) + 4x$

$= 1 - (2 + 1 - 3x + 3x) + 4x$

$= 1 - (3) + 4x$

$= -2 + 4x$

Activity 4

Discussion

(a) $x - 5 = y$ add 5 to both sides $x = y + 5$

(b) $3x - 3 = y$ add 3 to both sides, then divide by 3 $x = \frac{(y + 3)}{3} = \frac{y}{3} + 1$

(c) $\frac{x}{2} = 3y$ multiply both sides by 2 $x = 6y$

(d) $\frac{x}{7} - 4 = 2y$ multiply both sides by 7 $x - 28 = 14y$ add 28 to both sides $x = 14y + 28$

(e) $\frac{x}{2.5} = 10y$ multiply both sides by 2.5 $x = 25y$

(f) $2x - 12 = 2y$ add 12 to both sides $2x = 2y + 12$ divide both sides by 2 $x = y + 6$

Activity 5

Discussion

(a) $3x = 6$ so, $x = 2$

(b) $7x = 21$ so, $x = 3$

(c) $8x = 32$ so, $x = 4$

(d) $x + 3 = 9$ so, $x = 6$

(e) $x + 6 = 7$ so, $x = 1$

(f) $x + 7 = 11$ so, $x = 4$

Activity 6

Discussion

	Equation	Step 1	Step 2	Solution
(a)	$2x - 1 = 7$	$2x = 8$		$x = 4$
(b)	$5x - 8 = 2$	$5x = 10$		$x = 2$
(c)	$3x + 8 = 5$	$3x = -3$		$x = -1$
(d)	$3x - 8 = 5x - 20$	$-8 = 5x - 3x - 20$	$12 = 2x$	$x = 6$
(e)	$3(x + 1) = 9$	$3x + 3 = 9$	$3x = 6$	$x = 2$
(f)	$23 - x = x + 11$	$23 = 2x + 11$	$2x = 12$	$x = 6$
(g)	$2(x - 3) - (x - 2) = 5$	$2x - 6 - x + 2 = 5$	$x - 4 = 5$	$x = 9$
(h)	$\frac{4x}{5} = 12$	$4x = 60$		$x = 15$
(i)	$\frac{x}{3} = \frac{2}{5}$	$x = \frac{6}{5}$		$x = 1.2$
(j)	$\frac{x}{3} + \frac{x}{5} = 2$	$x + \frac{3x}{5} = 6$	$5x + 3x = 30$	$x = 3.75$
(k)	$\frac{x}{4} - \frac{x}{7} = 2$	$x - \frac{4x}{7} = 8$	$7x - 4x = 56$	$x = 18.7$ (to 1 decimal place)

Activity 7

Discussion

	Equations	Step 1: rearrange to make y the subject	Step 2: solve for x	Step 3: solve for y	Solution
(a)	$y = x + 10$	$y = x + 10$	$x + 10 = \frac{(2x + 5)}{3}$	$y = x + 10$	$x = -25$
	$3y = 2x + 5$	$y = \frac{(2x + 5)}{3}$	$3x + 30 = 2x + 5$	$y = -25 + 10 = -15$	$y = -15$
			$x = -25$		
(b)	$y = 4x$		$4x = 3x + 5$	$y = 4x = 20$	$x = 5$
	$y = 3x + 5$		$x = 5$		$y = 20$
(c)	$y = 7x + 4$	$y = 7x + 4$	$7x + 4 = \frac{(x + 7)}{3}$	$y = 7x + 4$	$x = -0.25$
	$3y = x + 7$	$y = \frac{(x + 7)}{3}$	$21x + 12 = x + 7$	$y = -1.75 + 4 = 2.25$	$y = 2.25$
			$20x = -5$		
			$x = -0.25$		
(d)	$y = 2x + 4$	$y = 2x + 4$	$2x + 4 = \frac{(x + 7)}{3}$	$y = 2x + 4$	$x = -1$
	$3y = x + 7$	$y = \frac{(x + 7)}{3}$	$6x + 12 = x + 7$	$y = -2 + 4 = 2$	$y = 2$
			$5x = -5$		
			$x = -1$		

5 Units, significant figures and scientific notation

Have you come across an assignment question or an exam question where you think everyone is setting out to make life as difficult as possible for you? You know which formula to use but none of the figures you have been given can be plugged in directly.

Here is a question from an exam paper on a technology course:

> *A simply supported square-section beam has a load of 5.0 kN hung from it centrally. The beam is constructed from solid aluminium with 50 mm × 50 mm cross-section and is 0.4 m in length between the end supports. What is the central deflection when the load is present? Take Young's Modulus to be 72 GN m^{-2}.*

Don't worry if this question doesn't make sense to you – it is purely an example to illustrate the use of scientific notation and significant figures.

Throughout this section we shall use the following symbols for the values referred to in the question.

Force	F
Length	l
Young's Modulus	E
Breadth	b
Depth	d
Deflection	Δ

The formula for central deflection, which we will be using later in this section, is

$$\Delta = \frac{Fl^3}{48EI}$$

Units

The first issues is that of units. The values are not given using a coherent system of units. The system used in science and engineering problems is the Système Internationale, usually called the SI system (Table 1).

Table 1 Some common SI units

SI unit	What it measures	Symbol
kilogram	mass	kg
metre	length	m
second	time	s
joule	energy	J
newton	force	N
kelvin	temperature	K

Because of the magnitude of values, it is sometimes appropriate to use another letter alongside that used to define the unit (Table 2). This makes calculations more manageable.

Table 2 Some common prefixes used with SI units

Prefix	What it stands for	Definition
m	milli	$\times 10^{-3}$
k	kilo	$\times 10^{+3}$
M	mega	$\times 10^{+6}$
G	giga	$\times 10^{+9}$

Now look at the units in the question. Only the distance between the supports is given in an SI unit. The remainder need to be converted. The kN needs to be converted to N, the mm need to be converted to m, and the GN m^{-2} to N m^{-2}. This necessitates a lot of multiplying and dividing by multiples of 10 and gives scope for getting the order of magnitude wrong.

Another point we would like you to note at this stage is that the data in the question is given to no more than two-figure accuracy. When we work through the calculation, we will find that the central deflection is 1.77777 m. Is it reasonable to quote to five decimal places? Surely we can't justify an accuracy of 1/10000 m when some of the data is accurate to only 1/10 m. What accuracy can we quote in the answer?

The ideas of significant figures and scientific notation enable us to deal with these issues systematically.

Significant figures

The number of significant figures in a number is found by counting all the digits from the first non-zero digit on the left.

Leading zeros are *not* significant figures. Trailing zeros are significant figures *only* when they occur after the decimal point.

Let's look at two examples.

(a) 765.430 has six significant figures. You start counting from the 7 which is the first non-zero digit on the left. The trailing zero is a significant figure; if it were not, it would not be necessary to include it.

(b) 0.04321 has four significant figures. The leading zeros are essential to give the magnitude of the number. The first non-zero digit on the left is 4 and counting the significant digits then gives us a total of four.

Example 1

Write the following values correct to the number of significant figures given beside each.

(a) 723.6792 (4 sig figs)

(b) 0.0763 (1 sig fig)

(c) 6382 (2 sig figs)

Discussion

(a) 723.6792

First count four digits from the left. This gives 723.6

Now look at the next digit.

This is 7 which indicates that the number is actually closer to 723.7 than 723.6

Rounding to four significant figures gives 723.7

(b) 0.0763

Counting from the first non-zero digit on the left, we have 0.07

Now examine the next digit.

This is 6 which indicates that the number is closer to 0.08 than 0.07

Rounding to one significant figure gives 0.08

(c) 6382

Counting from the left, the first two significant figures are 6 and 3

The next digit is 8 so we round up the 3 to 4

This gives a value of 6400 to two significant figures.

Activity 1

Now here are some exercises for you to try.

1 How many significant figures do these numbers have?

(a) 0.6540 (d) 700.302

(b) 760.45 (e) 1.9

(c) 10002 (f) 0.00029

2 Write each of these values to the number of significant figures stated in brackets.

(a) 0.00328 (2) (c) 0.79312 (4)

(b) 974.721 (3) (d) 1654.769 (4)

Discussion

1 The number of significant figures in each case is given below.

(a) 4 (d) 6

(b) 5 (e) 2

(c) 5 (f) 2

2 The solutions are given below.

(a) 0.0033 (c) 0.7931

(b) 975 (d) 1655

Back to the original problem

> *A simply supported square-section beam has a load of 5.0 kN hung from it centrally. The beam is constructed from solid aluminium with 50 mm × 50 mm cross-section and is 0.4 m in length between the end supports. What is the central deflection when the load is present? Take Young's Modulus to be 72 GN m^{-2}.*

Now let us identify the numerical data given in the original problem and convert it into SI units.

Symbol	Value from question	Value in SI units
F	5.0 kN	5.0×10^3 N
l	0.4 m	0.4 m
E	72 GN m^{-2}	72×10^9 N m^{-2}
b	50 mm	50×10^{-3} m
d	50 mm	50×10^{-3} m

The SI units could now be substituted into the formula for deflection, but they are not all in the same format. This is where the idea of scientific notation comes in.

Scientific notation

In order to make such a calculation simpler, we usually work in scientific notation. Scientific notation is really quite a simple idea, which allows us to write all of our numbers in the same format.

Scientific notation uses the format $a.b \times 10^c$

In other words, there is always *only one digit* in the range $1 \leq \text{digit} \leq 9$ (i.e. one digit in the range 1 to 9) before the decimal point regardless of the magnitude of the number.

So, for example, 674 is written as 6.74×10^2

58.4 is written as 5.84×10^1

0.0756 is written as 7.56×10^{-2}

Activity 2

Write the following numbers in scientific notation.

(a) 56432

(b) 0.00091

(c) 222

(d) 11.645

(e) 7543.46

(f) 0.054321

Discussion

(a) $56432 = 5.6432 \times 10^4$

The decimal point in 56432 lies after the final digit. It is not usual to include the decimal point when there are no digits following it. We have moved the decimal point four places to the left, which is the same as dividing by 10^4. In order to keep the magnitude of the number correct, we must now multiply by 10^4.

(b) $0.00091 = 9.1 \times 10^{-4}$

In this example, the decimal point has to be moved four places to the right, which is equivalent to multiplying by 10^4. We must divide the resultant number by 10^4, which is the same as multiplying by 10^{-4}.

(See Section 3 on indices for clarification if necessary.)

(c) $222 = 2.22 \times 10^2$

We have moved the decimal point two places to the left, thus dividing by 10^2. We must now multiply by 10^2 to maintain the correct value.

(d) $11.645 = 1.1645 \times 10^1$

In moving the decimal point one place to the left, we have divided by 10^1 and must now multiply by 10^1 to keep the original value.

(e) $7543.46 = 7.54346 \times 10^3$

The decimal point is moved three places to the left, hence we have divided by 10^3. To keep the original value we must then multiply by 10^3.

(f) $0.054321 = 5.4321 \times 10^{-2}$

To write this value in scientific notation, the decimal point is moved two places to the right, multiplying the value by 10^2. It is necessary to divide by 10^2 to keep the order of magnitude correct; this is the same as multiplying by 10^{-2}.

Back to the original problem

We already have a value for F in scientific notation: $F = 5.0 \times 10^3$. However, l, E, b and d are not in the correct format as they do not have one digit before the decimal point.

Let us look at l first. The decimal point needs to be moved one place to the right. If we move the decimal point one place to the right, we are multiplying the number by 10^1. We must then multiply the resulting value by 10^{-1} to ensure that the magnitude of the number is not changed.

$l = 0.4\ \text{m} = 4.0 \times 10^{-1}\ \text{m}$

If we look at the values for E, b and d we can see that there is more than one digit in front of the decimal point. So we have to move the decimal points to the left. If we move the decimal point one place to the left, we are dividing by 10^1 so we must then multiply by 10^1 to keep the magnitude of each value correct.

$E = 72 \times 10^9\ \text{N m}^{-2}$

$= 7.2 \times 10^1 \times 10^9\ \text{N m}^{-2}$

$= 7.2 \times 10^{10}\ \text{N m}^{-2}$

Remember: $10^a \times 10^b = 10^{a+b}$ (see Section 3 on indices for clarification if necessary).

The value for E is now written in scientific notation and SI units.

The values of b and d are identical and may be written as

$50 \times 10^{-3}\ \text{m}$

$= 5.0 \times 10^1 \times 10^{-3}\ \text{m}$

$= 5.0 \times 10^{-2}\ \text{m}$

Now we can produce a new set of values that are in both SI units and scientific notation.

F 5.0×10^{3} N

l 4.0×10^{-1} m

E 7.2×10^{10} N m^{-2}

b 5.0×10^{-2} m

d 5.0×10^{-2} m

Now we should be able to calculate an answer to the problem posed at the outset of this section.

You may not be familiar with the equation, but the procedures are the same regardless of the formula so don't worry if this is not your area of study.

The formula for deflection of a beam in this situation was given as

$$\Delta = \frac{Fl^3}{48EI}$$

We do not have a value for I. This entity is the second moment of area of the beam and is given by the formula

$$I = \frac{bd^3}{12}$$

Let us calculate I.

$$I = \frac{bd^3}{12} = \frac{(5.0 \times 10^{-2}) \times (5.0 \times 10^{-2})^3}{12}$$

$$= \frac{5.0 \times 10^{-2} \times (5.0)^3 \times (10^{-2})^3}{12}$$

$$= \frac{5 \times 125 \times 10^{-2} \times 10^{-6}}{12}$$

$$= \frac{625 \times 10^{-8}}{12}$$

Remember: $10^a \div 10^b = 10^{a-b}$ and $(10^a)^b = 10^{ab}$ (see Section 3 on indices for clarification if necessary).

As we are going to substitute this into the formula for deflection, it is best to leave the value for I in this format. This will lead to a more accurate answer.

Now substituting in the formula for deflection gives

$$\Delta = \frac{Fl^3}{48EI} = \frac{5.0 \times 10^3 \times (4.0 \times 10^{-1})^3}{48 \times 7.2 \times 10^{10} \times (625/12) \times 10^{-8}}$$

$$= \frac{5.0 \times (4.0)^3 \times 10^3 \times 10^{-3}}{48 \times 7.2 \times (625/12) \times 10^{10} \times 10^{-8}}$$

$$= \frac{5 \times 64}{4 \times 7.2 \times 625 \times 10^2}$$

$$= 0.0177777 \times 10^{-2}$$

$$= 1.77777 \times 10^{-4}$$

If we look at the number of digits following the decimal point and compare this to the data given in the question, we would appear to have a value that is more accurate than the data used to produce it. Rounding this value to two significant figures gives a value of 1.8×10^{-4} m. To give us an idea of what the deflection might look like, let us convert this to mm: 1.8×10^{-4} m = 0.18 mm, which is a very small deflection in the beam.

To help you grasp the procedure, we are going to work through another example.

Example 2

Vertical strings support two metal spheres so that they are touching. Sphere 1, which has a mass m_1 of 50 g, is pulled aside to the right until it reaches a height h_1 of 20 cm. It is then released and swings to undergo an elastic collision with sphere 2, which has a mass m_2 of 120 g.

(a) What is the velocity of the two spheres immediately after the collision?

(b) To what height do the two spheres swing just after the collision?

Assume g, the acceleration due to gravity, is 9.81 m s^{-2}.

Discussion

First, identify the numerical values and convert them into SI units and scientific notation.

Symbol	Value	Value in SI units	Value in scientific notation
m_1	50 g	50×10^{-3} kg	5.0×10^{-2} kg
g	9.81 m s^{-2}	9.81 m s^{-2}	9.81 m s^{-2}
h_1	20 cm	20×10^{-2} m	2.0×10^{-1} m
m_2	120 g	120×10^{-3} kg	1.20×10^{-1} kg

Initially both spheres have no velocity.

When sphere 1 is moved to the right it has potential energy, which is converted to kinetic energy just before the collision. This enables us to calculate the velocity of sphere 1 prior to the collision.

Let u_1 be the velocity of sphere 1 before the collision and u_2 the velocity of sphere 2 before the collision.

Conservation of energy gives us

$$m_1 g h_1 = \frac{m_1 u_1^2}{2}$$

So, $5.0 \times 10^{-2} \times 9.81 \times 2.0 \times 10^{-1} = \dfrac{5.0 \times 10^{-2} \times u_1^2}{2}$

Rearranging this equation gives

$$u_1^2 = \frac{2 \times 5 \times 10^{-2} \times 9.81 \times 2.0 \times 10^{-1}}{5.0 \times 10^{-2}}$$

$= 2 \times 9.81 \times 2 \times 10^{-1}$

Note: $10^{-3} \div 10^{-2} = 10^{(-3-(-2))} = 10^{-1}$

$= 4 \times 9.81 \times 10^{-1}$

$= 3.924$

$u_1 = \sqrt{3.924}$

So, $u_1 = 1.980908882$ m s^{-1}.

Conservation of momentum gives us

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

where v is the velocity of the spheres immediately after collision.

But since m_2 is at rest before the collision, $u_2 = 0$.

Hence

$5 \times 10^{-2} \times 1.980908882 = (5 \times 10^{-2} + 1.2 \times 10^{-1})v$

$5 \times 10^{-2} \times 1.980908882 = 1.7 \times 10^{-1} \times v$

$$v = \frac{5 \times 10^{-2} \times 1.980908882}{1.7 \times 10^{-1}}$$

$$= \frac{5 \times 1.980908882 \times 10^{-2} \times 10^{+1}}{1.7}$$

$= 5.826202595 \times 10^{-1}$

$v = 0.58$ m s^{-1} (to 2 significant figures).

It is good practice to work through calculations without rounding until the final value is arrived at. This avoids cumulative errors occurring.

The kinetic energy immediately after the collision is given by

$$\text{KE} = \frac{(m_1 + m_2)}{2} \times v^2$$

$$= \frac{5.0 \times 10^{-2} + 1.2 \times 10^{-1}}{2} \times (5.826202595 \times 10^{-1})^2$$

$$= \frac{0.5 \times 10^{-1} + 1.2 \times 10^{-1}}{2} \times (5.826202595)^2 \times (10^{-2})$$

$$= \frac{1.7 \times 10^{-1}}{2} \times (5.826202595)^2 \times (10^{-2})$$

$$= \frac{1.7}{2} \times (5.826202595)^2 \times (10^{-1} \times 10^{-2})$$

So, KE $= 28.85294117 \times 10^{-3}$ joules.

When the spheres swing to their maximum height, the velocity is zero and all the kinetic energy is converted into potential energy. Hence

$(m_1 + m_2)g\, h_2 = 28.85294117 \times 10^{-3}$

$1.7 \times 10^{-1} \times 9.81 \times h_2 = 28.85294117 \times 10^{-3}$

Rearranging to find h_2

$$h_2 = \frac{28.85294117 \times 10^{-3}}{1.7 \times 10^{-1} \times 9.81}$$

$$= \frac{28.85294117 \times 10^{-3} \times 10^{1}}{1.7 \times 9.81}$$

$= 1.730103806 \times 10^{-2}$

So, $h_2 = 1.7 \times 10^{-2}$ m.

Giving the value to 2 significant figures, $h_2 = 1.7$ cm.

We hope you have grasped the procedure now.

Procedure for solving problems involving SI units, scientific notation and significant figures

- Identify numerical values.
- Convert them to SI units.
- Write them in scientific notation.
- Substitute figures in formula.
- Separate into decimal numbers and powers of 10.
- Calculate the answer.
- Decide on the appropriate number of significant figures. This should not exceed the number of significant figures in the data given.
- Give value to appropriate number of significant figures.
- Convert into meaningful units for the example.

Now follow the above procedure to solve the next problem.

Activity 3

Two cars are stopped side by side at traffic lights. When the lights change to green both cars accelerate. The Jaguar XJS reaches a speed of 45 km per hr in 30 seconds while the Nissan Micra takes $2\frac{1}{2}$ minutes to reach the same speed.

- Assuming uniform acceleration in both cars, calculate how many kilometres each will have travelled in five minutes?
- What is the distance between them at this point?

Acceleration, a, is given by $v = u + at$, and distance travelled, s, is given by $ut + \frac{at^2}{2}$ where

v = final velocity

u = initial velocity

t = time.

Discussion

Identify numerical values from the question

Jaguar	initial velocity	$u = 0\ \text{m s}^{-1}$
	final velocity	v = 45 km per hr
	time	t = 30 s
Micra	initial velocity	$u = 0\ \text{m s}^{-1}$
	final velocity	v = 45 km per hr
	time	$t = 2\frac{1}{2}$ minutes
Final time	T = 5 minutes	

Convert to SI units and write in scientific notation

		Symbol value	Numerical	SI units	Scientific notation
Jaguar	Initial speed	u	0 m s^{-1}	0 m s^{-1}	0 m s^{-1}
	Final speed	v	45 km per hr	12.5 m s^{-1}	1.25×10^1 m s^{-1}
	Time	t	30 s	30 s	3.0×10^1 s
Micra	Initial speed	u	0 m s^{-1}	0 m s^{-1}	0 m s^{-1}
	Final speed	v	45 km per hr	12.5 m s^{-1}	1.25×10^1 m s^{-1}
	Time	t	$2\frac{1}{2}$ minutes	150 s	1.5×10^2 s
Final time		T	5 minutes	300 s	3.0×10^2 s

Substitute in formula, separate numbers and calculate

To find the acceleration, a, use $v = u + at$ which gives

$$a = \frac{v - u}{t}$$

(If you are not sure about rearranging equations, refer to Section 4.)

In both cases $u = 0$ m s^{-1}.
So, for the Jaguar

$$a = \frac{1.25 \times 10^1}{3.0 \times 10^1} = 0.416666666 \text{ m s}^{-2}$$

Acceleration for the Jaguar is 0.416666666 m s^{-2}.

For the Micra

$$a = \frac{1.25 \times 10^1}{1.5 \times 10^2} \text{ m s}^{-2}$$

Acceleration for the Micra is 0.083333333 m s^{-2}.

To find the distance travelled, use $s = ut + \frac{at^2}{2}$

In both cases

$u = 0$ m s^{-1}.

So, $s = \frac{at^2}{2}$

$t = 300$ s

$t^2 = 90000$

For the Jaguar

$$s = \frac{1}{2} \times 0.416666666 \times 90000 = 18750 \text{ m}$$

For the Micra

$$s = \frac{1}{2} \times 0.083333333 \times 90000 = 3750 \text{ m}$$

So the Jaguar has travelled 18.750 km and the Micra has travelled 3.750 km. The distance between them is 15 km.

All values are in km as the question required, and the distance between them is exactly 15 km when the values are not rounded during the calculation.

6 Basic trigonometry

We are going to look at some of the basics of trigonometry relating to right angle triangles. So the first question is, what is a right angle triangle?

It is a triangle in which one of the angles is 90˚, which is commonly referred to as a right angle. The sum of the angles in any triangle is 180˚. So if the other two angles are α and β as in Figure 1

$\alpha + \beta + 90 = 180^\circ$

$\alpha + \beta = 90^\circ$.

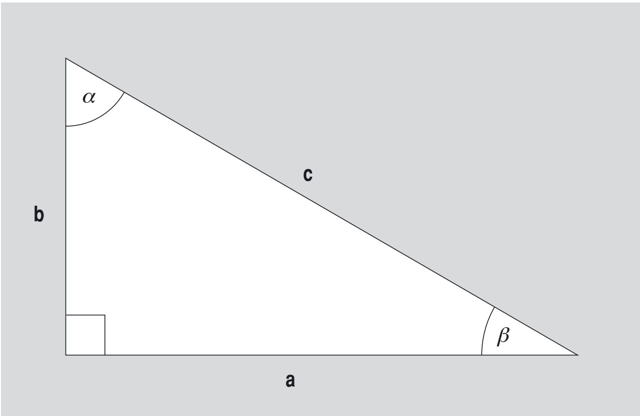

Figure 1 A right angle triangle

Look at the angle β in Figure 1. The side of the triangle opposite to this is labelled b and is called the 'opposite'. The side next to β is labelled a and is referred to as the 'adjacent'. The side labelled c is called the 'hypotenuse'. The hypotenuse is always the side opposite the right angle.

Pythagoras' Theorem states that the square of the length of the hypotenuse of a right angle triangle is equal to the sum of the squares of the lengths of the other two sides. Looking at the triangle in Figure 1

$c^2 = a^2 + b^2$

This formula, together with some knowledge of trigonometry enables us to calculate the angles and sides of the triangle in Figure 1 (given some other information).

Now look at your calculator. *A scientific calculator is essential if you are going to use any trigonometry*. Scientific calculators operate in two modes when dealing with angles. Before embarking on any of the examples please ensure that your calculator is in degree mode. Follow the instructions in your calculator manual to do this.

You will see three buttons, probably close to the screen and to the right hand side of the keypad, labelled sin, cos and tan. These are the functions we will be using and are called trigonometric (or 'trig') functions.

Sin = sine, cos = cosine, tan = tangent.

We can now define these functions for you, based on the angle β and the right angle triangle in Figure 1.

$$\sin \beta = \frac{\text{opposite } (b)}{\text{hypotenuse } (c)}$$

$$\cos \beta = \frac{\text{adjacent } (a)}{\text{hypotenuse } (c)}$$

$$\tan \beta = \frac{\text{opposite } (b)}{\text{adjacent } (a)}$$

There is an easy way to remember this: SOHCAHTOA.

$\text{Sin} = \frac{\text{O}}{\text{H}}$ $\text{Cos} = \frac{\text{A}}{\text{H}}$ $\text{Tan} = \frac{\text{O}}{\text{A}}$

If, for any right angle triangle, we know two sides or a side and an angle, then we can find the other side, sides or angle as required.

You might like to experiment with your calculator finding sin and cos for various angles. You should notice that sin and cos always lie between 0 and 1 if the input angle is between 0° and 90°.

Example 1

We are going to find the unknown values, α, a and c in Figure 2.

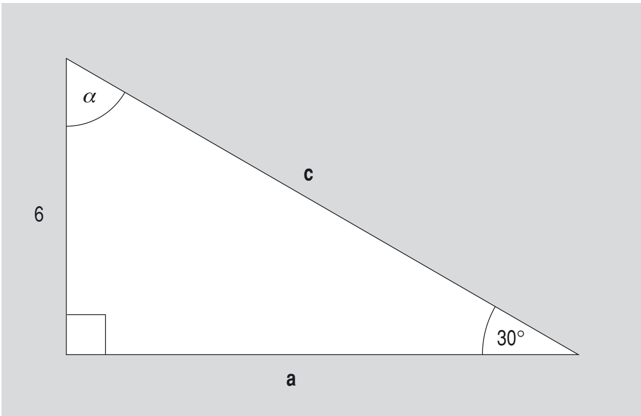

Figure 2 A right angle triangle for Example 1

Discussion

In Figure 2, the angle β is 30° and the side b is 6 cm.

The unknown angle α can be found using the fact that the sum of the internal angles of a triangle is 180°.

$\alpha = 180 - (90 + 30) = 180 - 120 = 60^\circ$

Now calculate a, which is adjacent to the angle β, which is 30°. We know the opposite side is 6 cm.

If you look at SOHCAHTOA, you will find that the function relating adjacent and opposite is tan.

$$\tan 30^\circ = \frac{6}{a}$$

$$a \tan 30^\circ = 6$$

$$a = \frac{6}{\tan 30^\circ} = 10.392304485$$

The side a is 10.392304485 cm.

The only remaining unknown is c and this we can find using Pythagoras' Theorem.

$c^2 = a^2 + b^2$

$c^2 = (10.39230485)^2 + 6^2$

$c^2 = 108 + 36 = 144$

$c = \sqrt{144} = 12$

The side c is 12 cm.

Using the rules for a triangle, some basic trigonometry and Pythagoras, and knowing the length of one side and the size of one angle, we have found the remaining angles and sides of the triangle.

Activity 1

Now try this one yourself. Find the unknown side b and the angles α and β of Figure 3.

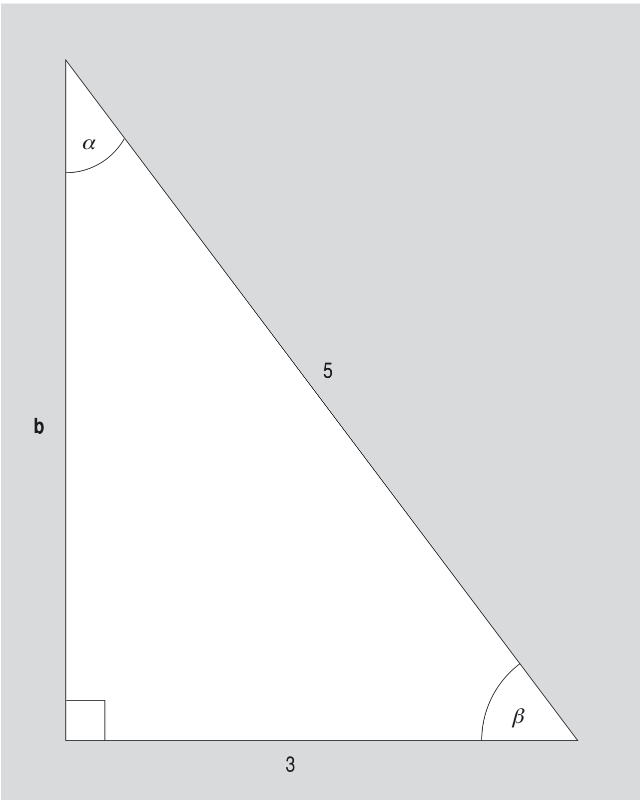

Figure 3 A right angle triangle for Activity 1

Discussion

This time we know the length of two sides. In relation to the angle β, they are the adjacent and the hypotenuse.

The trig function which relates these two sides is cos.

$$\cos \beta = \frac{3}{5} = 0.6$$

This tells us that β is the angle for which cos is 0.6; this can be written $\beta = \cos^{-1} 0.6$. We can now use a calculator to find β.

Look at your calculator again. You should see a key which has the letters *inv* above it. This key enables you to use the functions written above, rather than on, the other keys. Look at your sin, cos and tan keys again. Above each you should see what are known as the inverse functions, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.

To find $\cos^{-1} 0.6$, key in 0.6, press *inv* and then the cos key. The value which appears should be 53.13010236.

(Some calculators may require a slightly different methodology, so please check your calculator handbook.)

So, $\beta = 53.13^{\circ}$ (to 2 decimal places)

The angle α can be found in the same way as the third angle in Example 1.
$\alpha = 180 - (90 + 53.13) = 36.87^{\circ}$.

The opposite side b can be found using Pythagoras.

$c^2 = a^2 + b^2$

$5^2 = 3^2 + b^2$

$25 = 9 + b^2$

$25 - 9 = b^2$

$b^2 = 16$

$b = 4$

The length of side b is 4 cm.

Here are two more exercises for you to try.

Activity 2

In Figure 4, find the angles β and α and the length of side a.

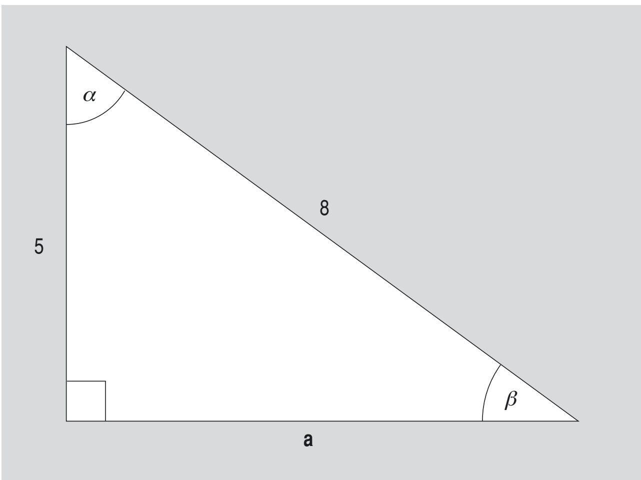

Figure 4 A right angle triangle for Activity 2

Discussion

$\sin \beta = \frac{5}{8}$

$\beta = \sin^{-1} \frac{5}{8}$

$\beta = 38.68^{\circ}$ (to 2 decimal places) and, since all the angles inside a triangle must add to 180º

$\alpha = 180 - (90 + 38.68)$

$\alpha = 51.32^{\circ}$ (to 2 decimal places)

From Pythagoras

$a^2 + 5^2 = 8^2$

$a^2 + 25 = 64$

$a^2 = 64 - 25$

$a^2 = 39$

$a = \sqrt{39}$

The length of side a is 6.24 cm (to 2 decimal places).

So we know all the sides and all the angles in the triangle.

Activity 3

Look at Figure 5 and find the sides a and b and the angle a.

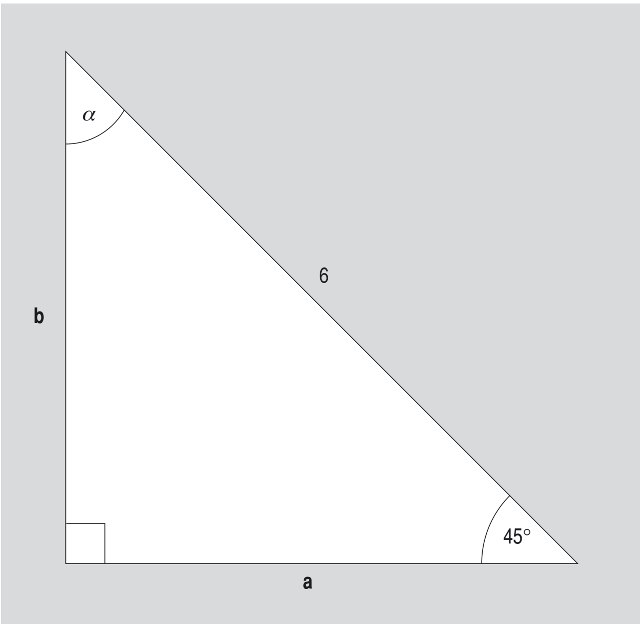

Figure 5 A right angle triangle for Activity 3

Discussion

This time we are starting with different information, but we still need to look for two known values related to one of the unknown values.

$\cos 45^{\circ} = \dfrac{a}{6}$

$6 \times \cos 45^{\circ} = a$

$6 \times 0.7071 = a$

side $a = 4.24$ cm

and

$\sin 45^{\circ} = \dfrac{b}{6}$

$6 \times \sin 45^{\circ} = b$

$6 \times 0.7071 = b$

side $b = 4.24$ cm

To find the other angle

$\alpha = 180 - (90 + 45)$

$\alpha = 180 - 135$

angle $\alpha = 45^{\circ}$

This is a particular type of triangle where side a = side b and there are two equal angles. This sort of triangle is known as an isosceles triangle.

What is the point of trigonometry?

One application that you might come across in applied mathematics, physics or technology courses concerns the resolution of forces.

Consider the case of a father pulling his two young children along on a sledge (Figure 6a). He has a rope attached to the sledge which makes an angle α with the ground. The force F in the rope is 50 N. If α is 25°, find the horizontal and vertical components of the force.

The force F acting in the rope may be considered as two component forces. We will call the horizontal component F_h and the vertical component F_v (Figure 6b).

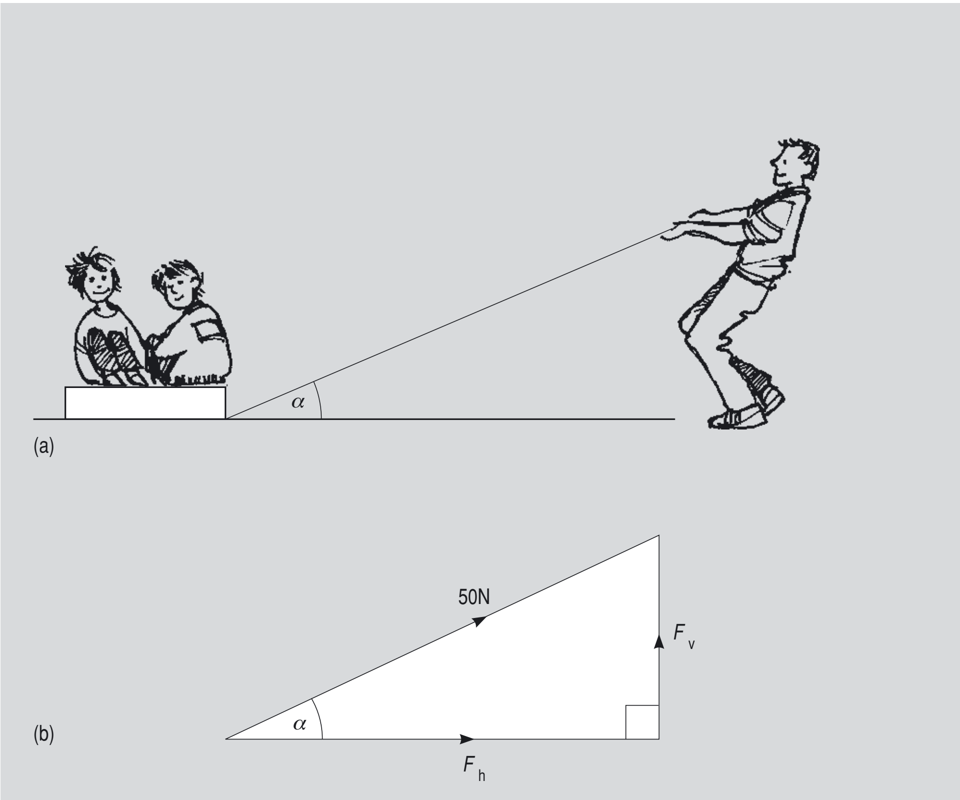

Figure 6 Using trigonometry to resolve forces

Looking at Figure 6b and applying the trigonometry from earlier in this section, we can say

$\cos \alpha = \frac{F_h}{50}$

and

$\sin \alpha = \frac{F_v}{50}$

Rearranging these two equations gives

$F_h = 50 \cos 25^\circ$

and

$F_v = 50 \sin 25^\circ$

So $F_h = 50 \times 0.09063$

$= 45.32$ N

The horizontal component of the force is 45.32 N.

$F_v = 50 \times 0.42262$

$= 21.13$ N

The vertical component of the force is 21.13 N.

We should find that $F_h^2 + F_v^2 = F^2$

Check using your calculator:

$45.32^2 + 21.13^2 = 50^2$

Now here is an activity for you.

Activity 4

Calculate the horizontal and vertical components of the force F if

(a) the angle α is reduced to 15°

(b) the angle α is increased to 45°

Discussion

(a) $F_h = 50 \cos 15^\circ$

$= 50 \times 0.9659$ N

$= 48.30$ N

The horizontal component of the force is 48.30 N.

$F_v = 50 \sin 15^\circ$

$= 50 \times 0.2588$ N

$= 12.94$ N

The vertical component of the force is 12.94 N.

(b) $F_h = 50 \cos 45^\circ$

$= 50 \times 0.7071$ N

$= 35.36$ N

The horizontal component of the force is 35.36 N.

$F_v = 50 \sin 45°$

$= 50 \times 0.7071$ N

$= 35.36$ N

The vertical component of the force is 35.36 N.

You can check these values using Pythagoras:

(a) $48.30^2 + 12.94^2 = 50^2$

and

(b) $35.36^2 + 35.36^2 = 50^2$

Now look at these values.

Can you identify any relationship between the angle and the components of the forces?

As the angle α increases, the horizontal component of the force decreases and the vertical component increases. In order to maximize the effectiveness of the force in terms of horizontal motion, the angle α should be kept as small as possible.

The activities in this section have been designed to help you to use trigonometry in problem solving. As you continue your studies you will encounter other applications.

7 Logarithms

The equation $2^3 = 8$ means that 3 is the index or the power to which we raise the number 2 to produce 8.

A logarithm is an index, and in this example, 3 is the logarithm of 8 to the base 2. We write this as

$\log_2 8 = 3$

These two equations are identical: $2^3 = 8$ and $\log_2 8 = 3$

They express the same fact in the language of logarithms.

> **Definition of a logarithm**
>
> If a number y can be written in the form a^x, then the index x is called the logarithm of y to the base a.
>
> If $y = a^x$
>
> then $\log_a y = x$

Since $2 = 2^1$, then $\log_2 2 = 1$

Similarly, $\log_3 3 = 1$, and $\log_{10} 10 = 1$, and as $a = a^1$, then $\log_a a = 1$

Also, since $1 = 2^0$ then $\log_2 1 = 0$

Similarly, $\log_3 1 = 0$ and $\log_{10} 1 = 0$, and as $1 = a^0$, then $\log_a 1 = 0$

Why do we use logarithms? Before the days of the electronic calculator, logarithms were used for multiplication and division and involved the use of log tables. Nowadays, logarithms are mainly used in integration or to find a linear function from an exponential one.

There are three rules which, with the definition of a logarithm, can be deduced from the rules for indices.

Rule 1 $\log_a xy = \log_a x + \log_a y$

Let $\log_a x = m$ and $\log_a y = n$

This means that $x = a^m$ and $y = a^n$

So $xy = a^m \times a^n = a^{(m+n)}$

Applying the definition of a logarithm gives

$\log_a xy = m + n = \log_a x + \log_a y$

For example, $\log_a 21 = \log_a 7 + \log_a 3$

Rule 2 $\log_a \frac{x}{y} = \log_a x - \log_a y$

Let $\log_a x = m$ and $\log_a y = n$ with $x = a^m$ and $y = a^n$

So $\frac{x}{y} = \frac{a^m}{a^n} = a^{(m-n)}$

Applying the definition of a logarithm gives

$\log_a \frac{x}{y} = \log_a x - \log_a y$

For example, $\log_a 7 = \log_a 14 - \log_a 2$

Rule 3 $\log_a x^r = r \log_a x$

Let $\log_a x = n$ with $x = a^n$

Raising each side to the power r gives

$x^r = (a^n)^r = a^{rn}$

Applying the definition of a logarithm gives

$\log_a x^r = rn = r \log_a x$

For example, $\log_{10} 1000 = \log_{10} 10^3 = 3 \log_{10} 10 = 3$ (because $\log_{10} 10 = 1$)

$\log_{10} 100 = 2$

$\log_{10} 1000000 = 6$

$\log_a \sqrt{3} = \log_a 3^{1/2} = \frac{\log_a 3}{2}$

Natural logarithms

The most frequently used bases for logarithms are 10 and the number 'e'. Rather like π, the irrational number 'e' occurs frequently in many branches of mathematics and its applications to science and engineering. Logarithms to base 10 are known as common logarithms and those to base 'e' are called natural or Napierian logarithms after the mathematician who discovered them.

Natural logarithms have the property that $\log_e e^x = x$.

We use natural logarithms to solve equations that contain the exponential function e^x where e is the irrational number 2.718281828 correct to ten significant figures.

When we work with logarithms to base 10 we drop the subscript and just write $\log x$. Natural logarithms are written as $\ln x$. This means that the rules for logarithms can be written as follows for natural logarithms.

Rule 1 $\ln xy = \ln x + \ln y$

Rule 2 $\ln \frac{x}{y} = \ln x - \ln y$

Rule 3 $\ln x^r = r \ln x$

Change of base

The final thing that we need to be able to do with logarithms is change the base.

Suppose we have $\log_a x$ and we want to find $\log_b x$.

Let $\log_b x = n$ so that $x = b^n$

Taking logarithms to base a gives

$\log_a x = \log_a (b^n) = n \log_a b$ (Rule 3)

Rearranging this gives

$$n = \frac{\log_a x}{\log_a b}$$

So, $\log_b x = \dfrac{1}{\log_a b} \times \log_a x$ (as $n = \log_b x$)

Thus, if we want to change between natural logarithms and logarithms to the base 10, we can use

$$\log x = \frac{\ln x}{\ln 10}$$

$$\ln x = \frac{\log x}{\log e}$$

Activity 1

(a) Simplify $\log 6 + \log 3 - \log 9$

(b) Write $4 \log x - \frac{1}{2} \log y + 3 \log z$ as a single logarithm

(c) Simplify $\log 64 \div \log 2$

(d) Simplify $(\log 27 - \log 9) \div \log 3$

(e) Find $\log_9 x$ given that $\log_3 x = 12$

Discussion

(a) $\log (6 \times 3 \div 9) = \log 2$

(b) $\log x^4 - \log y^{1/2} + \log z^3 = \log(x^4 \div \sqrt{y} \times z^3) = \log \dfrac{x^4 z^3}{\sqrt{y}}$

(c) $\log 2^6 \div \log 2 = \dfrac{6 \log 2}{\log 2} = 6$ $(2^6 = 64)$

(d) $\log (27 \div 9) \div \log 3 = \dfrac{\log 3}{\log 3} = 1$

(e) Changing the base gives $\log_9 x = \dfrac{\log_3 x}{\log_3 9} = \dfrac{12}{\log_3 3^2} = \dfrac{12}{2} = 6$

Note: $\log_3 3^2 = 2 \log_3 3$ and $\log_3 3 = 1$.

Activity 2

Solve the equations:

(a) $15 = 3e^{2x}$

(b) $2e^{-x/10} + 16 = 20$

Discussion

(a) $15 = 3e^{2x}$

Divide both sides by 3

$5 = e^{2x}$

$\ln 5 = \ln (e^{2x}) = 2x \ln e$

So, $\ln 5 = 2x$ as $\ln e = 1$

$x = \dfrac{\ln 5}{2}$

(b) $2e^{-x/10} + 16 = 20$

Subtract 16 from both sides

$2e^{-x/10} = 4$

Divide both sides by 2

$e^{-x/10} = 2$

$\ln(e^{-x/10}) = \ln 2$

So, $\dfrac{-x}{10} = \ln 2$

Multiply both sides by -10

$x = -10 \ln 2$

Activity 3

Express the following as the logarithm of one number:

(a) $2(1 - \log 2) + \log 3 - 3$

(b) $\frac{1}{2}\log (ab) - \frac{1}{2}\log a$

(c) $2 \log (ab) - 3 \log a$

Discussion

(a) Since $a = a^1$ then $\log_a a = 1$, so $\log 10 = 1$; also $1000 = 10^3$, so $\log 1000 = 3$. Thus

$2(1 - \log 2) + \log 3 - 3 = 2(\log 10 - \log 2) + \log 3 - \log 1000$

$= 2 \log 5 + \log 3 - \log 1000$ Rule 2

$= \log 5^2 + \log 3 - \log 1000$ Rule 3

$= \log 25 + \log 3 - \log 1000$

$= \log (25 \times 3 \div 1000)$ Rule 1, 2

$= \log 0.075$

If you multiply out the bracket first you will still get the same answer.

$2(1 - \log 2) + \log 3 - 3 = 2 - 2 \log 2 + \log 3 - 3$

$= \log 100 - \log 2^2 + \log 3 - \log 1000$

$= \log 100 - \log 4 + \log 3 - \log 1000$

$= \log (100 \div 4 \times 3 \div 1000)$*

$= \log 0.075$

*Note: The value of the bracket must be evaluated from left to right; if you multiply 4×3 first you will not get the correct answer. This is because

$$\log 100 - \log 4 + \log 3 - \log 1000 = \log \left[\frac{100}{4} \times \frac{3}{1000}\right]$$

(b) $\frac{1}{2} \log (ab) - \frac{1}{2} \log a = \frac{1}{2} (\log ab - \log a)$

$= \frac{1}{2} \log b$ Rule 2

$= \log \sqrt{b}$

(c) $2 \log (ab) - 3 \log a = \log (ab)^2 - \log a^3$ Rule 3

$= \log a^2b^2 - \log a^3$ Rule 2

$= \log \frac{b^2}{a}$

Activity 4

(a) Solve the equation $5^{-2x} = 625^{3/4}$

(b) From the formula $s = \frac{1}{k} \ln \left(\frac{L}{L - V^2}\right)$, show that $V^2 = L(1 - e^{-ks})$

Discussion

(a) Take logarithms of both sides

$\log(5^{-2x}) = \log 625^{3/4}$

$-2x \log 5 = \frac{3}{4} \log 625$

$= \frac{3}{4} \log 5^4$

$= 4 \times \frac{3}{4} \log 5$

So, equating both sides and dividing by log 5

$-2x = 3$

$x = -1.5$

(b) Multiply both sides by k

$ks = \ln \left(\frac{L}{(L - V^2)}\right)$

From the definition of a logarithm

$e^{ks} = \frac{L}{(L - V^2)}$

Rearranging gives

$L - V^2 = \frac{L}{e^{ks}}$

$= Le^{-ks}$

Multiply through by -1

$V^2 - L = -Le^{-ks}$

So, $V^2 = L - Le^{-ks}$

$= L (1 - e^{-ks})$

8 Further reading and sources of help

Where to get more help with using and interpreting tables, graphs, percentages, and with other aspects of numerical work.

Further reading

- *The Good Study Guide* by Andrew Northedge, published by The Open University, 2005, ISBN 0 7492 59744.
- *The Sciences Good Study Guide*, by Andrew Northedge, Jeff Thomas, Andrew Lane, Alice Peasgood, published by The Open University, 1997, ISBN 0 7492 3411 3.

  More mathematical and science-based than The Good Study Guide. Chapter titles are: 'Getting started', 'Reading and making notes', 'Working with diagrams', 'Learning and using mathematics', 'Working with numbers and symbols', 'Different ways of studying', 'Studying with a computer', 'Observing and experimenting', 'Writing and tackling examinations'. There are 100 pages of maths help on: calculations, negative numbers, fractions, decimals, percentages, approximations and uncertainties, powers and roots, scientific notation, formulas and algebra, interpreting and drawing graphs, perimeters, areas and volumes.
- *Countdown to Mathematics Volumes 1 & 2*, by Lynne Graham and David Sargent, published by Addison-Wesley, Slough, 1981, ISBN 0 201 13730 5 and ISBN 0 201 13731 3 respectively.

  This is a useful next stage after any of the above. Volume 1 includes an introduction to algebra. Volume 2 covers equations, trigonometry, indices and logarithms.
- *Basic Mathematics* (Teach Yourself Educational S.) by Alan Graham, 2003, published by Teach Yourself, ISBN 0 3408 59857.

  A book aimed at a more general (i.e. non-OU) audience in two parts entitled 'Understanding the basics' and 'Maths in action'.
- *Simple Statistics: A course book for the social sciences*, by Frances Clegg, published by Cambridge University Press, 1990, ISBN 0 521 28802 9.
- *Algebra* (Teach Yourself Educational S.) by P. Abbott and Hugh Neill, 2003, published by Teach Yourself, ISBN 0 3408 67434.

  An elementary introduction which covers the algebra required for applications of maths to engineering etc.
- *The Complete A-Z Mathematics Handbook*, 2nd edition, by John Berry, Ted Graham, Jenny Sharp and Elizabeth Berry, published by Hodder and Stoughton, London, 2000, ISBN 0 340 78030 4.

  The ideas, terms and concepts of mathematics are clearly explained. A useful reference book.
- *Tapping into Mathematics with the TI-83 Graphics Calculator*, by Barry Galpin and Alan Graham, published by Addison Wesley Longman Ltd, 1997, ISBN 0 201 17547 9.

  This book teaches you how to use a graphics calculator and covers some basic mathematics and statistics. However, if you have studied MU120 you will already have received this material.

Sources of further help

The Open University

Your Regional Centre may know of local courses. For example, some back-to-study courses have a mathematical element to them.

Local colleges and schools

The local newspaper is a source of reference here, or your local library. Alternatively, most schools and colleges nowadays have evening or daytime courses that are open to adult learners. Many of them will have an advice point, so that you can telephone or drop in to discuss what you are looking for. A number of open learning centres are also a source of self-assessment tests and open learning materials.

LearnDirect

This is a telephone line that was set up to help adults to find out about local provision of further and higher education courses. The number is 0800 100 900. Lines are open 9 a.m. to 9 p.m. Monday to Friday, 9 a.m. to 12 noon on Saturdays. Calls are free. There is also a comprehensive web site at http://www.learndirect.co.uk

Scottish Learning Network

The Scottish Learning Network at http://www.scottish-learning-network.co.uk is a gateway to information, guidance, assessment and on-line education and training opportunities in Scotland.